

Higher twist light-cone distribution amplitudes of vector mesons in QCD

Kazuhiro Tanaka^{a,*}

^aDepartment of Physics, Juntendo University, Inba-gun, Chiba 270-1606, Japan

We present a systematic study of twist-3 light-cone distribution amplitudes of vector mesons in QCD, which is based on conformal expansion. A complete set of distribution amplitudes is constructed for ρ , ω , K^* and ϕ mesons, which satisfies all (exact) equations of motion and constraints from conformal expansion.

Amplitudes of hard (light-cone dominated) exclusive processes in QCD are expressed by the factorization formulae, which separate the short-distance dynamics from the long-distance one[1]. For those processes producing a (light) vector meson V ($= \rho, \omega, K^*, \phi$) in the final state, like exclusive semileptonic or radiative B decays ($B \rightarrow V e \nu$, $B \rightarrow V + \gamma$) and the hard electroproduction of vector mesons ($\gamma^* + N \rightarrow V + N'$), a long-distance part involving the final vector meson is given by the vacuum-to-meson matrix element of the nonlocal light-cone operators $\langle 0 | \bar{\psi}(z) \Gamma \lambda^i \psi(-z) | V \rangle$, corresponding to universal nonperturbative quantity called the light-cone distribution amplitudes (DAs). (z_μ is a light-like vector $z^2 = 0$, and λ^i and Γ denote various flavor and Dirac matrices. We do not show the gauge phase factor connecting the quark and the antiquark fields.)

The DAs of higher twist are essential for systematic study of preasymptotic corrections to hard exclusive amplitudes, and are also interesting theoretically because they contain new and direct information on hadron structure and the dynamics of QCD.

Here we will explicitly consider the “chiral-odd” DAs of the charged ρ meson with momentum P_μ and polarization vector $e_\mu^{(\lambda)}$ ($P^2 = m_\rho^2$, $e^{(\lambda)} \cdot e^{(\lambda)} = -1$, $P \cdot e^{(\lambda)} = 0$). The “chiral-even” DAs can be treated similarly[2]; and extension to other vector mesons is straightforward (see below). The relevant quark-antiquark DAs are defined with the chirality-violating Dirac matrix structures $\Gamma = \{\sigma_{\mu\nu}, 1\}$ as

$$\begin{aligned} \langle 0 | \bar{u}(z) \sigma_{\mu\nu} d(-z) | \rho^-(P, \lambda) \rangle &= i f_\rho^T \left[\left(e_{\perp\mu}^{(\lambda)} P_\nu - e_{\perp\nu}^{(\lambda)} P_\mu \right) \int_0^1 du e^{i\xi P \cdot z} \phi_\perp(u, \mu^2) \right. \\ &\quad \left. + (P_\mu z_\nu - P_\nu z_\mu) \frac{e^{(\lambda)} \cdot z}{(P \cdot z)^2} m_\rho^2 \int_0^1 du e^{i\xi P \cdot z} h_\parallel^{(t)}(u, \mu^2) \right], \end{aligned} \quad (1)$$

$$\langle 0 | \bar{u}(z) d(-z) | \rho^-(P, \lambda) \rangle = -i \left(f_\rho^T - f_\rho \frac{m_u + m_d}{m_\rho} \right) m_\rho^2 (e^{(\lambda)} \cdot z) \int_0^1 du e^{i\xi P \cdot z} h_\parallel^{(s)}(u, \mu^2). \quad (2)$$

In (1), we neglect the Lorentz structures corresponding to the twist-4 terms for simplicity (see Ref.[2] for the complete expressions). Our Lorentz frame is chosen as $P \cdot z =$

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P^+z^- . The nonlocal operators on the l.h.s. is renormalized at scale μ . We set $\xi \equiv u - (1 - u) = 2u - 1$, and f_ρ and f_ρ^T are the usual vector and tensor decay constants as $\langle 0|\bar{u}(0)\gamma_\mu d(0)|\rho^-(P, \lambda)\rangle = f_\rho m_\rho e_\mu^{(\lambda)}$ and $\langle 0|\bar{u}(0)\sigma_{\mu\nu}d(0)|\rho^-(P, \lambda)\rangle = if_\rho^T(e_\mu^{(\lambda)}P_\nu - e_\nu^{(\lambda)}P_\mu)$. We wrote $e_\mu^{(\lambda)} = e_{\parallel\mu}^{(\lambda)} + e_{\perp\mu}^{(\lambda)}$ with $e_{\parallel\mu}^{(\lambda)} = [P_\mu - z_\mu m_\rho^2/(P \cdot z)](e^{(\lambda)} \cdot z)/(P \cdot z)$, so that the DAs with the subscripts \parallel and \perp describe longitudinally and transversely polarized ρ mesons, respectively. ϕ_\perp is of twist-2, while $h_{\parallel}^{(t)}$ and $h_{\parallel}^{(s)}$ are of twist-3. These DAs are dimensionless functions and describe the probability amplitudes to find the ρ in a state with quark and antiquark, which carry the light-cone momentum fractions u and $1 - u$, respectively, and have a small transverse separation of order $1/\mu$.

The quark-antiquark-gluon DAs can be defined similarly; they are of twist-3 and higher. There exists one chiral-odd DA of twist-3, which is given by

$$\begin{aligned} \langle 0|\bar{u}(z)\sigma^{\mu\nu}z_\nu gG_{\mu\eta}(vz)z^\eta d(-z)|\rho^-(P, \lambda)\rangle \\ = (P \cdot z)(e^{(\lambda)} \cdot z)f_{3\rho}^T m_\rho \int_0^1 \mathcal{D}\underline{\alpha} e^{-iP \cdot z(\alpha_u - \alpha_d + v\alpha_g)} \mathcal{T}(\underline{\alpha}, \mu^2), \end{aligned} \quad (3)$$

where $G_{\mu\eta}$ is the gluon field strength tensor, $\mathcal{D}\underline{\alpha} \equiv d\alpha_d d\alpha_u d\alpha_g \delta(1 - \alpha_d - \alpha_u - \alpha_g)$, and $\underline{\alpha}$ denotes the set of three light-cone momentum fractions: α_d (quark), α_u (antiquark), and α_g (gluon). $f_{3\rho}^T$ is the three-body tensor decay constant[2], so that \mathcal{T} is dimensionless and is conveniently normalized as $\int \mathcal{D}\underline{\alpha}(\alpha_d - \alpha_u) \mathcal{T}(\underline{\alpha}) = 1$. The three-particle DA \mathcal{T} describes a higher component in the Fock-state wave function with additional gluon.

The basis of DAs defined above is overcomplete due to the constraints from the QCD equations of motion. Using $(i \not{D} - m_q)q(x) = 0$ ($q = u, d$), we obtain

$$\begin{aligned} \frac{1}{2}x^\nu \frac{\partial}{\partial x_\mu} \bar{u}(x) [\gamma_\mu, \gamma_\nu]_\pm d(-x) = \int_{-1}^1 dv v^{(1\mp 1)/2} \bar{u}(x) \sigma^{\mu\nu} x_\nu gG_{\mu\eta}(vx) x^\eta d(-x) \\ - \frac{x^\nu}{2} \frac{\partial}{\partial y_\mu} \left\{ \bar{u}(x+y) [\gamma_\mu, \gamma_\nu]_\mp d(-x+y) \right\} \Big|_{y \rightarrow 0} + i(m_u \pm m_d) \bar{u}(x) \not{x} d(-x), \end{aligned} \quad (4)$$

where $[\gamma_\mu, \gamma_\nu]_\pm \equiv \gamma_\mu \gamma_\nu \pm \gamma_\nu \gamma_\mu$. In the light-cone limit $x^2 \rightarrow 0$, the vacuum-to-meson matrix elements of (4) yield a system of integral equations between two- and three-particle DAs defined above. We note that the total derivative term induces mixing between $h_{\parallel}^{(t)}$ and $h_{\parallel}^{(s)}$. We can solve[2] these coupled integral equations in a form ($i = s, t$)

$$h_{\parallel}^{(i)}(u, \mu^2) = \int_0^1 dv K_{WW}^{(i)}(u, v) \phi_\perp(v, \mu^2) + \int_0^1 \mathcal{D}\underline{\alpha} K_g^{(i)}(u, \underline{\alpha}) \mathcal{T}(\underline{\alpha}, \mu^2), \quad (5)$$

where $K_X^{(i)}$ ($X = WW, g$) are independent of μ . We omit the quark mass correction term for simplicity. The first term on the r.h.s. is the twist-2 contribution and thus the analogue of the Wandzura-Wilczek piece of the nucleon structure function $g_2(x, Q^2)$.

To analyze (5), we introduce the conformal partial wave expansion in the $m_q \rightarrow 0$ limit[2,3]. The conformal expansion of light-cone DAs is analogous to the partial wave expansion of wave functions in standard quantum mechanics (QM). In conformal expansion, the invariance of massless QCD under conformal transformations substitutes the rotational symmetry in QM. In QM, the purpose of partial wave decomposition is to separate angular degrees of freedom from radial ones (for spherically symmetric potentials).

All dependence on the angular coordinates is included in spherical harmonics which form an irreducible representation of the group $O(3)$, and the dependence on the single remaining radial coordinate is governed by a one-dimensional Schrödinger equation. Similarly, the conformal expansion of DAs in QCD aims to separate longitudinal degrees of freedom from transverse ones. All dependence on the longitudinal momentum fractions is included in terms of certain orthogonal polynomials which form irreducible representations of the so-called collinear subgroup of the conformal group, $SL(2, R)$. The transverse-momentum dependence (the scale-dependence) is governed by simple renormalization group equations: the different partial waves, labeled by different “conformal spins”, behave independently and do not mix with each other. Since the conformal invariance of QCD is broken by quantum corrections, mixing of different terms of the conformal expansion is only absent to leading logarithmic accuracy. Still, conformal spin is a good quantum number in hard processes, up to small corrections of order α_s^2 .

The conformal expansion of the DAs on the r.h.s. of (5) reads

$$\phi_{\perp}(u, \mu^2) = 6u\bar{u} \sum_{n=0}^{\infty} a_n(\mu^2) C_n^{3/2}(\xi); \quad \mathcal{T}(\underline{\alpha}, \mu^2) = 360\alpha_d\alpha_u\alpha_g^2 \sum_{k,l=0}^{\infty} \omega_{kl}(\mu^2) J_{kl}(\alpha_d, \alpha_u), \quad (6)$$

where $\bar{u} = 1 - u$, $\xi = 2u - 1$, $\alpha_g = 1 - \alpha_d - \alpha_u$, and $C_n^{3/2}$ and J_{kl} are particular Gegenbauer and Appell polynomials[2]. Using orthogonality relations of these polynomials, the expansion coefficients are expressed by matrix elements of local conformal operators: a_n and ω_{kl} are given by local operators of conformal spin $j = n + 2$ and $j = k + l + 7/2$, respectively (three-particle conformal representations are degenerate). Thanks to conformal symmetry, $K_X^{(i)}$ of (5) are resolved into the superposition of the terms of definite conformal spin. As a result, $h_{\parallel}^{(i)}$ are given by the conformal expansion, where each expansion coefficient is expressed in terms of a_n and ω_{kl} with the corresponding spin j .

Now all DAs are expressed, order by order in the conformal expansion, by independent matrix elements a_n and ω_{kl} . From (5) and (6), the μ^2 -dependence of the DAs is governed by that of $a_n(\mu^2)$ and $\omega_{kl}(\mu^2)$. This is determined by the renormalization of the corresponding local conformal operators, and is worked out in the leading logarithmic approximation[2,4]. The results indicate that the relevant anomalous dimensions increase as $\sim \ln j$. This means that only the first few conformal partial waves contribute at sufficiently large scales. Therefore, the truncation of the conformal expansion at some low order provides a useful and consistent approximation of the full DAs.

We take into account the partial waves with $j \leq 9/2$, where the terms with $n \leq 2$ and $k + l \leq 1$ are retained in the first and second equations of (6). Correspondingly, we get[2]

$$\begin{aligned} h_{\parallel}^{(s)}(u) &= 6u\bar{u} \left[1 + a_1\xi + \frac{1}{4}a_2(5\xi^2 - 1) \right] + 35u\bar{u}\zeta_3(5\xi^2 - 1) \\ &+ 3\delta_+(3u\bar{u} + \bar{u} \ln \bar{u} + u \ln u) + 3\delta_-(\bar{u} \ln \bar{u} - u \ln u), \end{aligned} \quad (7)$$

$$\begin{aligned} h_{\parallel}^{(t)}(u) &= 3\xi^2 + \frac{3}{2}a_1\xi(3\xi^2 - 1) + \frac{3}{2}a_2\xi^2(5\xi^2 - 3) + \frac{35}{4}\zeta_3(3 - 30\xi^2 + 35\xi^4) \\ &+ \frac{3}{2}\delta_+(1 + \xi \ln \bar{u}/u) + \frac{3}{2}\delta_-\xi(2 + \ln u + \ln \bar{u}). \end{aligned} \quad (8)$$

Here $\zeta_3 = f_{3\rho}^T/(f_{\rho}^T m_{\rho})$, and we incorporated the quark mass corrections proportional to $\delta_{\pm} = f_{\rho}(m_u \pm m_d)/(f_{\rho}^T m_{\rho})$. The results for other vector mesons ω , K^* and ϕ are

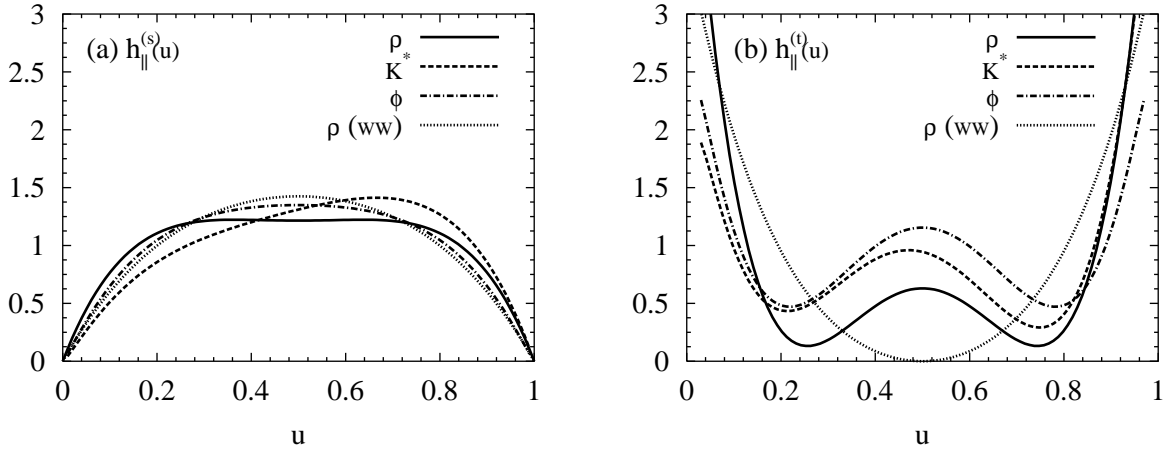


Figure 1. Two-particle twist-3 DAs for the ρ , K^* and ϕ mesons: (a) $h_{\parallel}^{(s)}(u)$ of (7); (b) $h_{\parallel}^{(t)}(u)$ of (8). “ ρ (WW)” denotes the “Wandzura-Wilczek” type contribution given by the first three terms of (7) and (8) for the case of the ρ meson.

obtained by trivial substitutions. We emphasize that these results provide a consistent set of DAs involving a minimum number of independent nonperturbative parameters. These parameters are calculated from QCD sum rules taking into account SU(3)-breaking effects[2]. The resulting DAs are plotted in Fig. 1. The comparison between the solid and dotted curves shows that the three-particle contributions (the term with ζ_3 in (7), (8)) are important and broaden the distributions. On the other hand, the quark mass effects are not so large except for the end-points $u \rightarrow 0$ and $u \rightarrow 1$ for $h_{\parallel}^{(t)}(u)$.

In conclusion, we have developed a powerful framework for hard exclusive processes, which allows to express the higher twist DAs in a model-independent way by a minimum number of nonperturbative parameters. Combined with estimates of the nonperturbative parameters, this leads to model building of the DAs consistent with all QCD constraints. Our formalism is applicable to arbitrary twist[5] and other light mesons[6], and our results are immediately applicable to a range of phenomenologically interesting processes[7].

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